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An important question about rock climbing

Giancarlo Reali†§ and Ledo Stefanini‡

† INFM, Dipartimento di Elettronica, Facoltà di Ingegneria, Università di Pavia, Via Ferrata 1, 27100 Pavia, Italy
‡ CAI, Club Alpino Italiano, Sezione di Mantova and Diploma in Ingegneria delle Risorse e dell'Ambiente, Sede di Mantova dell'Università di Pavia, Via Frattini 7, 46100 Mantova, Italy

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Abstract. The important question of how to limit the non-uniform force of the rope acting on a falling climber is discussed from the point of view of elementary physics.

Riassunto. L'importante problema di come limitare le forza non uniforme che agisce su uno scaletore in caduta è discussa dal punto di vista della fisica elementare.

1. Introduction

In a recent paper [1], circulated on the Internet, John Wylie, a physicist member of the Alpine Club of Canada, deals with a few interesting problems encountered in the practice of rock climbing. The most important question raised in [1] relates to the way one belays one's falling partner by means of a climbing rope, and Wylie describes an old (static) belaying technique that was used for many years up to the 1960s. The basic idea is that the climber, after free falling a distance $2d$, is decelerated by the action of the rope. If the second climber (the belayer) stops the rope at the anchor, the deceleration is non-uniform and occurs over a distance Δx which depends upon the elasticity of the rope and the total length of the rope being stretched.

If the rope is too stiff, serious damage to the climber can occur due to the jerk acting upon him; otherwise, if the jerk is too small, the rope behaves very much like rubber, which also involves dangerous situations for the climber.

Modern belaying techniques are based, instead, on dynamic belaying which is not dealt with by Wylie: as the fall occurs and the rope starts stretching the belayer releases a certain extra length of the rope which moves through the anchor device under approximately constant tension, dissipating by friction a considerable fraction of the fall kinetic energy of the climber and keeping the jerk and the rubber effect within controllable and safe limits.

Of course, this is something one can see instinctively but we want to consider it in some detail in this paper, since it represents a practical and useful example of a non-uniform force experienced by a falling body.

§ E-mail address: reali@dragon.ian.pv.cnr.it

2. Leader falls, follower belays

Figure 1(a) shows two climbers held by a rope, in which the first one acts as the leader while the second is the belayer.

While the leader, of mass m (80 kg) is climbing, the follower stops at some belay station, where he is secured to the rock, spaced no farther than the length of the rope. Let us state two hypotheses:

H1: each anchor (the one at the belay station and the intermediate ones) is assembled by way of protections or pitons attached firmly to the rock (also, the rope twists should not be wrapped around sharp edges or anything else that could damage the rope);

H2: when the leader slips off the rock (figure 1(b)), the belayer at the station can stop the rope instantaneously.

Most often, hypothesis H1 is more a hope than a reality, while as far as H2 is concerned, we will demonstrate why it is safer to act so that it is not perfectly realized.

Supposing for the moment that the above conditions are verified; if the leader falls, his initial gravity potential energy is first converted into kinetic energy and then again into the elastic potential energy of the rope. Indeed, a climbing rope [2] is a dynamic object which is able to stretch under tension and absorb momentum over relatively long times. This is a very important feature, because otherwise an (almost) inextensible rope would exert a fatal injury on the falling climber due to

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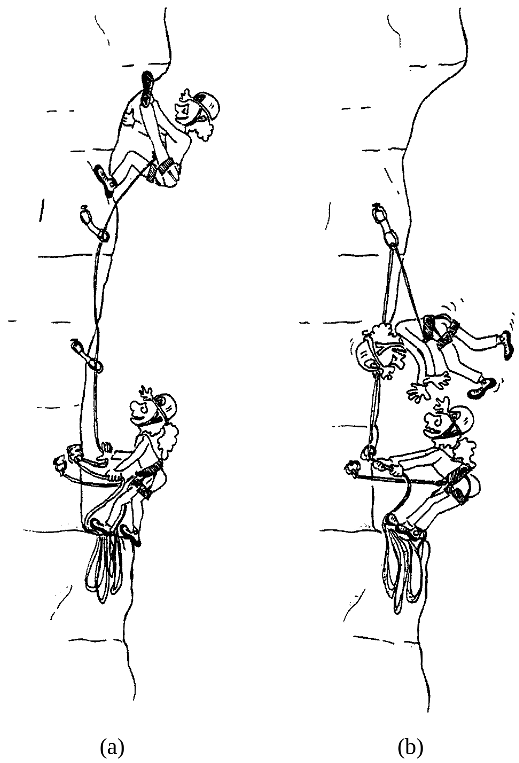

Figure 1. (a) Leader and follower held by the climbing rope; (b) slipping by the leader, the follower tries to stop the rope.

the huge applied elastic tension T, developed during the extremely short deceleration time or, equivalently, the extremely short stopping distance, $\Delta x \approx 0$:

$$T\,\Delta x = mg(2d) \tag{1}$$

where $2d$ is the falling distance.

We would like to investigate the rope performances and the way of improving the operational techniques in order to get a better compromise between strength and damping action of the equipment.

Let us call d the distance that separates the leader from the last anchor. In the case of slipping off the rock, the leader will fall a distance $2d + \Delta x_{\mathrm{max}}$, where Δx_{max} indicates the maximum stretching length of the rope, due to its elasticity. For the rope, Hooke's law is given by

$$F_{\mathrm{el}} = -M\frac{\Delta x}{L}, \tag{2}$$

where F_{el} is the force developed by the rope, L its length between the leader and the follower, and M the elasticity or rope modulus. $M = EA$, E being Young's modulus and A the rope cross sectional area. The rope elastic energy is given by the relation

$$E_{\mathrm{el}} = \frac{1}{2}\frac{M}{L}\Delta x^2, \tag{3}$$

and energy conservation requires that

$$mg(2d + \Delta x_{\mathrm{max}}) = \frac{1}{2}\frac{M}{L}\Delta x_{\mathrm{max}}^2. \tag{4}$$

From (4) we obtain Δx_{max}:

$$\Delta x_{\mathrm{max}} = \frac{L}{\kappa}[1 + \sqrt{1 + 2\kappa f}], \tag{5}$$

where we have set $\kappa = M/mg$. The 'fall factor' $f = 2d/L$ is the ratio between the falling distance and the rope length from the leader to the belayer, and its value is at most equal to two (when no protection is placed between them). The peak elastic force is obtained by inserting (5) into (2):

$$F_{\mathrm{el,max}} = -M\frac{\Delta x_{\mathrm{max}}}{L} = -mg[1 + \sqrt{1 + 2\kappa f}]. \tag{6}$$

In all rock climbing courses it is pointed out, unfortunately without much success, that the force does not depend on the released rope length L but only on the fall factor f. Let us remind readers that this result is obtained with the understanding that hypothesis H2 is satisfied.

In the mountain's reference frame, the climber is acted upon by a total force

$$F = mg - \frac{M}{L}\Delta x \tag{7}$$

during the deceleration phase, while during his free-fall of length $2d$ he does not feel any force at all, and only the force $-(M/L)\Delta x$ when the rope starts stretching. It has been found experimentally that a human body cannot support accelerations greater than $5g$ for a fraction of a second (even if much greater accelerations have been applied, under controlled conditions, to particularly trained persons). From this experimental result it follows that

$$mg\left[1 + \sqrt{1 + 2\left(\frac{M}{mg}\right)f}\right] \leq m5g, \tag{8}$$

which implies a value of the rope moduli limited by (in the worst case, $f = 2$)

$$M = 3.75mg \tag{9}$$

which for a climber of 80 kg means $M = 3$ kN. Note that, at the peak allowable acceleration, the rope tension is $T = m5g = 4$ kN and the corresponding rope stretch is as much as $\Delta x/L = 4/3 \approx 133\%$, a true rubber! This is clearly an unacceptable performance, as is easily verified with typical numbers: if the belayer released a 10 m rope, and the last protection was set at the belayer, the leader's fall would involve a 20 m free-fall ($f = 2$) plus 13 m of elastic stretching of the rope. A large part of the accumulated elastic energy is then released back in the rebound, which projects the climber up again (and so on a few times), and more likely causes the leader to hit the rock, one of the most common climbing injuries.

Ropes with greater moduli are actually employed in practice. The UIAA (Union Internationale des

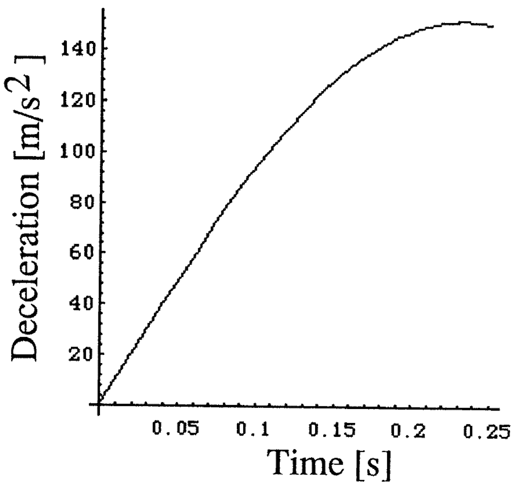

Figure 2. Deceleration during the fall, suffered by the leader.

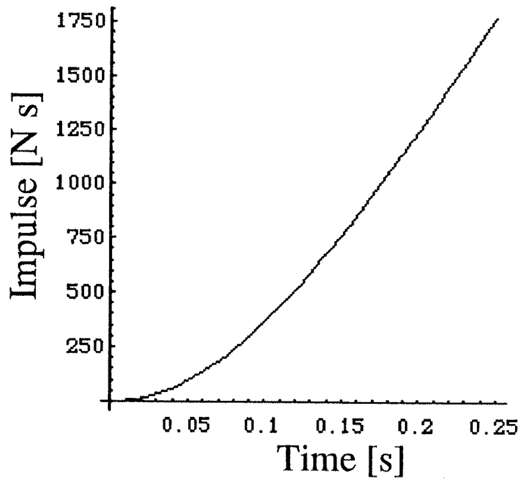

Figure 3. Time history of the impulse acting on the leader during the fall.

Associationes Alpinistiques) allows rope moduli up to 40 kN, definitely much stiffer than the previously quoted figure, corresponding to greater tensions, from (6) up to 12 kN, but to a much smaller rope stretching of $\Delta x/L \approx 0.3 \approx 30\%$. The above example in this case would involve again a free-fall of 20 m plus 3 m of stretched rope, with little rubber effect. It is important to recognise explicitly the dynamical nature of the figures stated above, which differ from the static figures, taken by hanging an 80 kg mass on one end of the rope while the other end is secured to an anchor. From $mg = M\Delta x/L$, we get for the static stretching $(\Delta x/L)_s = 27\%$ and $(\Delta x/L)_s = 2\%$, respectively, for the two values of M considered.

On the other hand, stiffer ropes could appear dangerous to the climber because of the greater deceleration imparted. We shall see why this is not the case if H2 is partially relaxed.

Let us first study in some detail the dynamics of the falling climber from the time the rope elastic response sets in. Using (7) in Newton's second equation, we have

$$\frac{\mathrm{d}^2\Delta x}{\mathrm{d}t^2} + \omega^2\Delta x = g \qquad (10)$$

where $\omega = \sqrt{(M/mL)} = \sqrt{\kappa(g/L)}$. Integrating (10) with initial conditions

$$\Delta x(t=0) = 0 \qquad v(t=0) = \sqrt{2g(2d)}, \qquad (11)$$

we obtain the solution

$$\Delta x(t) = \frac{L}{\kappa}[1 - \cos\omega t + \sqrt{2\kappa f}\sin\omega t]. \qquad (12)$$

The velocity and the acceleration come easily from the time derivation of (12). The velocity $v(t) = \sqrt{(gL/\kappa)}[\sin\omega t + \sqrt{2\kappa f}\cos\omega t]$ is zero for $\tan\omega t = -\sqrt{2\kappa f}$, that is at the time

$$t = \sqrt{\frac{L}{\kappa g}}[\pi - \arctan(\sqrt{2\kappa f})] \qquad (13)$$

which represents the deceleration time. For $L = 10$ m and $f = 2$, $t_2 \approx 0.23$ s.

In the climber's reference frame, the only force felt by him is the time variable elastic force,

$$F_{\mathrm{el}}(t) = -mg[1 - \cos\omega t + \sqrt{2\kappa f}\sin\omega t]. \qquad (14)$$

The corresponding deceleration with time is shown in figure 2.

We note that the deceleration on the falling climber goes up to approximately $15g$ in this case, far greater than $5g$ in the previous case. In reality, to judge the chance of a climber's injury it is not only important to look at the acceleration but also at the time the body is suffering it. The impulse, that is the integrated force over the time it is acting, is the quantity that conveys the needed information. A brief discussion and a dramatic picture showing why this is so appears in Halliday–Resnick–Walker's '*Fundamentals of Physics*' [3].

The integration of (14) from zero to t gives

$$I(t) = -\frac{mg}{\omega}[\omega t - \sin\omega t - \sqrt{2\kappa f}(\cos\omega t - 1)] \qquad (15)$$

and its behaviour is shown in figure 3.

We observe that if we assume the impulse as a measure of injury, the chance of injury is greater during the final part of the deceleration phase. Qualitatively, this happens because the belay point is stopped and the rope–climber system can exert on it a continuously increasing tension, to which it responds with an opposite tension applied on the climber. This is why it is safe that H2 is not strictly verified.

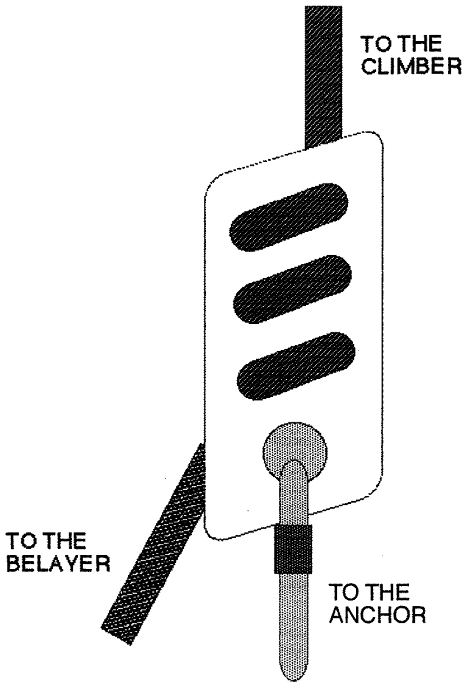

Figure 4. How the dissipator is made: the rope is passed through the holes several times.

If part of the work of the tension could be dissipated by friction of the rope on some non-destructive device, the acceleration and the impulse could be maintained to within a safe level. The experienced belayer knows the way to perform a safe stopping-release operation of the rope: the rope is knotted to a carabiner (an anchor device, [2]) by means of a so-called 'Italian knot', discovered about twenty years ago, whose peculiarity is that it does not lock the rope but lets it move through the carabiner with a very high friction.

More effective devices have been developed since that can be used instead of the carabiner and the Italian knot. One of these, named a 'dissipator', is schematically shown in figure 4.

This device for the first half of its job behaves as a resistance force amplifier. In the case of a fall, the rope is stopped until a certain tension threshold is reached, and the belayer, holding one end of the rope by means of the dissipator, is able to balance a much higher force applied at the other end. The way the dissipator holds the rope until the threshold is reached can be approximately understood by referring to the model of a rough cylinder around which the rope is wrapped several times. The tension T_0 at one end withstands a much greater tension T at the other end according to the relation $T = T_0 \exp(\mu_s \vartheta)$: for the dissipator, we usually have five half-turns of the rope on its way through (figure 4). If the threshold tension is set to $5mg = 4000$ N and the applied

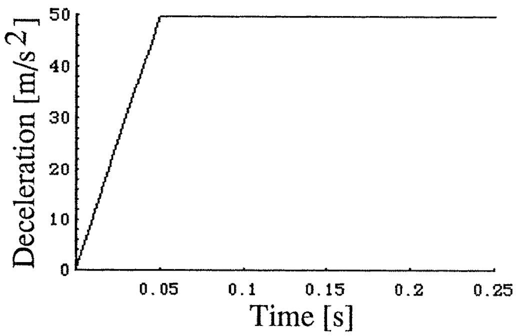

Figure 5. Ideal time response of the dissipator.

force by the belayer is 500 N, a rather modest static friction coefficient of $\mu_s \approx \log(4000/500)/5\pi \approx 0.12$ is required to hold the rope before its ultimate release.

From that instant on, the rope begins moving with high friction into the dissipator, keeping the tension (and the deceleration) approximately constant at the threshold value. This is how it is possible to prevent the climber from undergoing the final, most dangerous deceleration phase. This behaviour is shown qualitatively in figure 5.

Because of the dynamics, the climber suffers a maximum tension of

$$M\frac{x}{L} \le 5mg \qquad (16)$$

which can alternatively be put as

$$g[1 - \cos\omega t + \sqrt{2\kappa f}\sin\omega t] \le 5g. \qquad (17)$$

From (16) we get the rope stretching at the tension threshold when $M = 40$ kN: $\Delta x/L \approx 0.10 = 10\%$, an acceptable value. The solution of (17) provides, instead, the time that this value is reached, which turns out to be approximately $t_1 \approx 0.05$ s. So, in the case when H2 is verified and keeping all the previous conditions constant, the climber will suffer a greater than $5g$ acceleration for $\Delta t \equiv t_2 - t_1 = 0.18$ s for a 20 m fall ($L = 10$ m, $f = 2$), a rather long time.

However, if the dissipator is used, after $t_1 = 0.05$ s the maximum acceleration is reached and the rope will suffer a small stretch of only 10%. The velocity of the falling climber at t_1 is $v_1 \approx 20$ m s^{-1}. After this time the acceleration will remain approximately constant and equal to $5g$, and the climber will reach a velocity $v_3 = 0$ after an elapsed time $t_3 = t_1 + v_1/5g \approx 0.45$ s, which means that the falling time is longer in this case by $t_3 - t_2 = 0.27$ s. During this time the belayer will have supplied $\Delta x = v_1(t_3 - t_1) - \frac{1}{2}5g(t_3 - t_1)^2 \approx 4$ m of rope, letting it move through the dissipator. (Usually this is much less, and a compromise is reached by releasing only a few metres of spare rope.)

How much work is done by friction as the rope is running through the holes of the dissipator? From

Newton's second law,

$$mg - F_{\text{friction}} = ma = -m(5g) \tag{18}$$

so

$$W = F_{\text{friction}}\, \Delta x = 6mg\, \Delta x \approx 19.2 \text{ kJ} \tag{19}$$

which represents the work needed to maintain the falling acceleration within the safety limit of $5g$. Note that this is a crucial point: releasing more rope at no energy price is not enough to make the fall less dangerous and, indeed, as the only result that would be of no relevance for the climber at all. The momentum has in some way to be damped on some device other than the falling climber, and this is how the dissipator (or equivalent device) performs the second half of its job.

References

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[3] Halliday D, Resnick R and Walker J 1993 *Fundamentals of Physics* (New York: Wiley) ch 2